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$\Delta I = 1/2$ and ϵ'/ϵ in large-N_c QCD

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Abstract: We present new results for the matrix elements of the Q_6 and Q_4 penguin operators, evaluated in a large-N_c approach which incorporates important $\mathcal{O}(N_c^2 \frac{n_f}{N_c})$ unfactorized contributions. Our approach shows analytic matching between short- and long-distance scale dependences within dimensional renormalization schemes, such as $\overline{\mathrm{MS}}$. Numerically, we find that there is a large positive contribution to the $\Delta I = 1/2$ matrix element of Q_6 and hence to the direct CP-violation parameter ε'/ε. We also present results for the $\Delta I = 1/2$ rule in $K \to \pi\pi$ amplitudes, which incorporate the related and important "eye-diagram" contributions of $\mathcal{O}(N_c^2 \frac{1}{N_c})$ from the Q_2 operator (i.e. the penguin-like contraction). The results lead to an enhancement of the $\Delta I = 1/2$ effective coupling. The origin of the large unfactorized contributions which we find is discussed in terms of the relevant scales of the problem.

Keywords: 1/N Expansion, Kaon Physics, QCD, Chiral Lagrangians.

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Contents

1. Introduction

The so called penguin operators in Particle Physics have a long history. Their existence was first pointed out by the ITEP group [1] who showed that in the process of integrating out heavy fields in the electroweak theory of the Standard Model, in the presence of QCD interactions, there appear new local four-quark operators, like e.g. (summation over quark colour indices within brackets is understood):

$$Q_4 = 4 \sum_{q=u,d,s} (\bar{s}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu d_L) \qquad \text{and} \qquad Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L)\,, \tag{1.1}$$

besides the two conventional operators

$$Q_2 = 4(\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L) \qquad \text{and} \qquad Q_1 = 4(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L)\,, \tag{1.2}$$

which had been considered up to then [2, 3]. These four-quark operators, modulated by their appropriate Wilson coefficients $C_i(\mu)$, which are functions of the masses of the fields which have been integrated out and the scale μ of whatever renormalization scheme has been used to carry out this integration, are part of the effective hamiltonian which describes the weak interactions of quarks at intermediate energies of a few GeV,[1]

$$\mathcal{H}_{\text{eff}}(\Delta S=1)=\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{us}^{*}V_{ud}\sum_{i=1}^{10}C_{i}(\mu)Q_{i}(\mu)\,. \tag{1.3}$$

The ITEP group also claimed that matrix elements of the Q_6 penguin operator were particularly important and could be at the origin of the experimentally well established $\Delta I=1/2$ enhancement in $K\to\pi\pi$ transitions. However, the dynamical mechanism they proposed was shown to violate Current Algebra Ward identities [5]–[8]. In fact, in the approximation where the Q_6 operator factorizes into a product of bilinear quark densities, the $K\to\pi\pi$ matrix elements turn out to be too small to explain the $\Delta I=1/2$ enhancement.

In the meantime, it was also shown that the Wilson coefficient $C_6(\mu)$ acquires a large imaginary part, as a result of the integration of the heavy t and b quarks and the flavour mixing structure of the CKM matrix in the Standard Model. As a result, the evaluation of $K\to\pi\pi$ matrix elements of the Q_6 operator, besides its potential contribution to $\Delta I=1/2$ transitions, has also become of utmost importance, as an ingredient for a plausible understanding of the observed size of the CP-violation parameter ϵ'/ϵ in the Standard Model.

The effective lagrangian which describes $|\Delta S|=1$ transitions, like $K\to\pi\pi$ and $K\to\pi\pi\pi$, in the presence of electromagnetic interactions and to lowest order in chiral perturbation theory (χPT), $\mathcal{O}(p^0)$ and $\mathcal{O}(p^2)$ in this case, has the following structure :

$$\mathcal{L}_{\text{eff}}^{\Delta S=1}=-\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{ud}V_{us}^{*}\left[\mathbf{g}_{\underline{8}}\mathcal{L}_{8}+\mathbf{g}_{\underline{27}}\mathcal{L}_{27}+e^{2}\mathbf{g}_{\underline{\mathrm{ew}}}\,\mathrm{tr}\left(U\lambda_{L}^{(32)}UQ_{R}\right)\right], \tag{1.4}$$

where

$$\mathcal{L}_{8}=\sum_{i=1,2,3}(\mathcal{L}_{\mu})_{2i}\;(\mathcal{L}^{\mu})_{i3}\qquad\text{and}\qquad\mathcal{L}_{27}=\frac{2}{3}(\mathcal{L}_{\mu})_{21}\;(\mathcal{L}^{\mu})_{13}+(\mathcal{L}_{\mu})_{23}\;(\mathcal{L}^{\mu})_{11}\,. \tag{1.5}$$

with

$$\mathcal{L}_{\mu}=-iF_{0}^{2}\;U(x)^{\dagger}D_{\mu}U(x)\,, \tag{1.6}$$

and

$$\lambda_{L}^{(32)}=\delta_{i3}\delta_{j2}\,,\qquad Q_{L}=Q_{R}=Q=\mathrm{diag}\left(\frac{2}{3},-\frac{1}{3},-\frac{1}{3}\right). \tag{1.7}$$

The pion decay coupling constant F_0 is the one in the chiral limit where the quark masses u, d, s are neglected ($F_0\simeq 87\,\mathrm{MeV}$). The matrix field U collects the Goldstone fields of the spontaneously broken chiral symmetry of the QCD lagrangian with three massless flavours, and $D_\mu U$ denotes the covariant derivative: $D_\mu U=\partial_\mu U-ir_\mu U+iUl_\mu$, in the presence of external chiral sources l_μ and r_μ of left- and right-handed currents. Notice that the octet term proportional to $\mathbf{g}_{\underline{8}}$ induces pure $\Delta I=1/2$ transitions, while the term proportional to $\mathbf{g}_{\underline{27}}$ induces both $\Delta I=1/2$ and $\Delta I=3/2$ transitions. The coupling constants $\mathbf{g}_{\underline{8}}$, $\mathbf{g}_{\underline{27}}$ and $\mathbf{g}_{\underline{\mathrm{ew}}}$ encode the dynamics of the integrated degrees of freedom in the chiral limit.

[1] See e.g. the lectures of A. Buras in ref. [4], where earlier references can also be found.

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These include, not only the heavy quark flavours, but also the hadronic light flavour states (resonances) other than the Goldstone fields explicitly present in the χPT lagrangian.

The coupling constant $\underline{\mathbf{g}_8}$ has both a real part (relevant for the $\Delta I = 1/2$ rule) and an imaginary part (which induces direct CP violation). The main purpose of this work is to present an evaluation of the contributions to both ϵ'/ϵ and $\Delta I = 1/2$ induced by the QCD penguin operators in eq. (1.1), as well as from the so called *eye-like configurations* of the Q_2 operator in eq. (1.2). We shall do this within the framework of the $1/N_c$ expansion. The methodology[2] is the same as the one which has been applied to other calculations of similar low-energy observables reported elsewhere [11]–[16]. It allows for important improvements with respect to earlier calculations within the framework of the $1/N_c$ expansion, reported in ref. [17] (at leading order) and in refs. [18]–[22] (at next-to-leading order).

The paper is organized as follows. Section 2 collects the general formula from which the bosonizations of the penguin operators can be obtained. Section 3 gives the results for the usual factorized contributions. Section 4 contains a detailed discussion of the calculation of the unfactorized contributions of $\mathcal{O}(N_c n_f)$. Section 5 contains the corresponding analytic results for the coupling constant $\underline{\mathbf{g}_8}$ with both the factorized and unfactorized contributions. The phenomenological implications for the $\Delta I = 1/2$ rule and for ε'/ε are presented in sections 6 and 7 respectively. Finally in section 8 we present a discussion of our results and conclusions.

2. Bosonization of the penguin operators

The Dirac operator $\mathcal{D}_\chi$ of the QCD lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} + i\bar{q}\mathcal{D}_\chi q\,, \tag{2.1}$$

in the presence of external chiral sources l_μ, r_μ, $\mathcal{M}$ and $\mathcal{M}^\dagger$, is defined as follows:

$$\mathcal{D}_\chi = \gamma^\mu(\partial_\mu + iG_\mu) - i\gamma^\mu \left[l_\mu \frac{1-\gamma_5}{2} + r_\mu \frac{1+\gamma_5}{2} \right] + i\left(\mathcal{M}\frac{1-\gamma_5}{2} + \mathcal{M}^\dagger \frac{1+\gamma_5}{2} \right), \tag{2.2}$$

where G_μ denotes the gluon matrix field, and $G^{(a)}_{\mu\nu}$ the gluon field strength tensor ($a = 1, \ldots, N_c^2 - 1$). The bosonization of the penguin operators Q_4 and Q_6, to $\mathcal{O}(N_c^2)$ and $\mathcal{O}(N_c)$, is then formally defined by the functional integrals [23]

$$\begin{aligned}\langle \mathbf{Q}_4(x) \rangle \;=&\; 4\,{\rm Tr}\,\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta l_\mu(x)_{3j}}\,{\rm Tr}\,\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta l^\mu(y)_{j2}} + \\ &+ 4\int d^4y \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)}\;{\rm Tr}\left(\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta l_\mu(x)_{3j}} \mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta l^\mu(y)_{j2}} \right),\end{aligned} \tag{2.3}$$

and

$$\begin{aligned}\langle \mathbf{Q}_6(x) \rangle \;=&\; -8\,{\rm Tr}\;\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta\mathcal{M}^\dagger(x)_{3j}}\,{\rm Tr}\;\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta\mathcal{M}(y)_{j2}} + \\ &+ 8\int d^4y \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)}\;{\rm Tr}\left(\mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta\mathcal{M}^\dagger(x)_{3j}} \mathcal{D}_\chi^{-1}(-i)\frac{\delta\mathcal{D}_\chi}{\delta\mathcal{M}(y)_{j2}} \right),\end{aligned} \tag{2.4}$$

[2]See e.g. refs. [9, 10] for recent reviews.

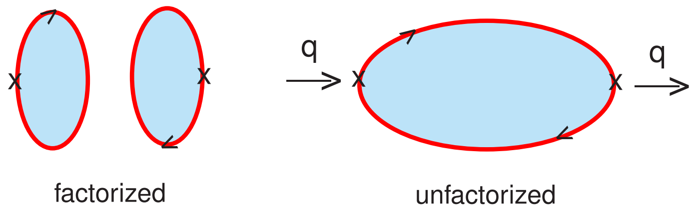

Figure 1: Factorized and unfactorized contributions corresponding to eqs. (2.3) and (2.4). The crosses correspond to the explicit variation of the Dirac propagator with respect to external sources (terms like $\frac{\delta \mathcal{D}_\chi}{\delta \mathcal{M}(y)_{j2}}$ in eq. (2.4)); the solid lines represent quarks, in the presence of *soft* external chiral sources, propagating in a *planar* gluon background.

where the trace Tr here also includes the functional integration over the *planar* gluonic configurations which leave the quarks at the edge [24, 25]. The first terms in eqs. (2.3) and (2.4) correspond to the *factorized* contributions, illustrated in figure 1, which are $\mathcal{O}(N_c^2)$; while the second terms in eqs. (2.3) and (2.4) correspond to the *unfactorized* $\mathcal{O}(N_c)$ contributions, also illustrated in figure 1. These *unfactorized* contributions involve integrals over the incoming q momenta, which are *regularization* dependent. For consistency, they have to be defined in the same renormalization scheme as those of the corresponding Wilson coefficients. With p the conjugate momentum operator, in the absence of the external chiral sources, the full quark propagator in x space is given by the expression

$$(x|\frac{1}{\mathcal{D}_\chi}|y) = (x|\frac{i}{\not{p} + \gamma^\alpha \left[l_\alpha \frac{1-\gamma_5}{2} + r_\alpha \frac{1+\gamma_5}{2} \right] - \mathcal{M}\frac{1-\gamma_5}{2} + \mathcal{M}^\dagger \frac{1+\gamma_5}{2}}|y), \tag{2.5}$$

where

$$(x|\not{p}|y) = \gamma^\mu \left[i\frac{\partial}{\partial x^\mu} - G_\mu \right] \delta(x-y)\,. \tag{2.6}$$

The bosonization of four-quark operators in χPT is then obtained via an appropriate chiral expansion in powers of the l_α, r_α, $\mathcal{M}$ and $\mathcal{M}^\dagger$ external sources in the propagators.

Let us recall that the spectrum of hadronic states which can contribute to these Green's functions in QCD in the large-N_c limit [24, 25] consists of an infinite number of narrow stable meson states which are flavour nonets. In the real world, however, the physical hadronic states have widths, which are subleading in $1/N_c$. The fact that the ratios of widths to masses are of $\mathcal{O}(1/N_c)$ is at the very basis of the phenomenological success of the large-N_c approach. Indirectly, this also suggests that in calculating physical observables, the contributions which stem from narrow states are the ones which may be potentially important. Notice that in the case of the bosonization of four-quark operators, the large-N_c QCD spectrum of narrow states contributes to the *factorized* as well as to the *unfactorized* patterns. Consequently one might already expect the *unfactorized* piece to be sizeable. Our calculation shows that this expectation is confirmed and the *unfactorized* contributions from these penguin operators turn out to be *larger* than those from the *factorized* terms. We shall later argue that this result becomes rather natural when formulated in terms of the different hadronic scales involved.

3. The factorized contributions

Let us first discuss the factorized contributions from the Q_6 operator. The operators $(\bar{s}_L q_R)$ and $(\bar{q}_R d_L)$ are Noether densities of the QCD lagrangian in eqs. (2.1) and (2.2):

$$
\begin{aligned}
(\bar{s}_L q_R) &= \bar{q}(x)\lambda_{3j}\frac{1+\gamma_5}{2}q(x) \equiv D_{\bar{s}q}(x) = -\frac{\delta\mathcal{L}_{\rm QCD}(x)}{\delta\mathcal{M}^\dagger(x)_{3j}} \\
(\bar{q}_L d_R) &= \bar{q}(x)\lambda_{j2}\frac{1-\gamma_5}{2}q(x) \equiv G_{\bar{q}d}(x) = -\frac{\delta\mathcal{L}_{\rm QCD}(x)}{\delta\mathcal{M}(x)_{j2}}\,.
\end{aligned}
\tag{3.1}
$$

On the other hand, the QCD effective lagrangian which describes the strong interactions of Goldstone particles at low energies is given, in $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, by a string of terms

$$
\begin{aligned}
\mathcal{L}_\chi = &\frac{1}{4}F_0^2\left\{\mathrm{tr}(D_\mu U D^\mu U^\dagger) + 2B\,\mathrm{tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)\right\} + \\
&+2BL_5\,\mathrm{tr}[D_\mu U^\dagger D^\mu U(\mathcal{M}^\dagger U + U^\dagger\mathcal{M})] + \cdots + \\
&+L_3\,\mathrm{tr}(D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U) + iL_9\,\mathrm{tr}(F_R^{\mu\nu}D_\mu U D_\nu U^\dagger + F_L^{\mu\nu}D_\nu U^\dagger D_\nu U) + \cdots\,,
\end{aligned}
\tag{3.2}
$$

where the first line gives the well known $\mathcal{O}(p^2)$ terms, and only those terms of the $\mathcal{O}(p^4)$ lagrangian [27] which will be needed in this paper have been explicitly written out. Notice that we are considering a framework in which the massive η' particle has been integrated out; while, strictly speaking, in a large-N_c formulation one should consider a $\mathrm{U}(3)_L\times\mathrm{U}(3)_R$ description of the chiral lagrangian with an explicit η_0-singlet Goldstone particle. We have checked that for the observables discussed in this paper, and at the level of the $\mathcal{O}(n_f/N_c)$ approximation that we are retaining, there is no phenomenological difference between the two formulations.

The bosonization of the Noether densities $D_{\bar{s}q}(x)$ and $D_{\bar{q}d}(x)$ can then be readily obtained as variations of the chiral lagrangian in eq. (3.2) with respect to the same external sources:

$$
D_{\bar{s}q}(x) = -\frac{\delta\mathcal{L}_\chi(x)}{\delta\mathcal{M}^\dagger(x)_{3j}} \Rightarrow 2B\,\mathrm{tr}\,\lambda_{3j}\left\{\frac{1}{4}F_0^2 U(x) + L_5 U D_\mu U^\dagger D^\mu U + \cdots\right\} + \mathcal{O}(p^4)\,, \tag{3.3}
$$

$$
G_{\bar{q}d}(x) = -\frac{\delta\mathcal{L}_\chi(x)}{\delta\mathcal{M}\ (x)_{j2}} \Rightarrow 2B\,\mathrm{tr}\,\lambda_{j2}\left\{\frac{1}{4}F_0^2 U^\dagger(x) + L_5 D_\mu U^\dagger D^\mu U U^\dagger + \cdots\right\} + \mathcal{O}(p^4)\,, \tag{3.4}
$$

where, among the $\mathcal{O}(p^2)$ contributions, we have only written the term induced by the L_5 coupling of the $\mathcal{O}(p^4)$ chiral lagrangian because it is the only term which contributes to the factorized bosonization of the Q_6 operator. Indeed, although the two densities $D_{\bar{s}q}(x)$ and $D_{\bar{q}d}(x)$ start with terms of $\mathcal{O}(p^0)$, their product — because of the fact that the U matrix is unitary — only starts at $\mathcal{O}(p^2)$ in the chiral expansion and, because of the flavour structure, this product can only depend on the L_5 coupling.

On the other hand, the factorized contribution from the Q_4 operator is rather straightforward. With

$$
\begin{aligned}
(\bar{s}_L\gamma^\mu q_L) &= \bar{q}(x)\lambda_{3j}\gamma^\mu\frac{1-\gamma_5}{2}q(x) \equiv L^\mu_{\bar{s}q}(x) = \frac{\delta\mathcal{L}_{\rm QCD}(x)}{\delta l_\mu(x)_{3j}}\,, \\
(\bar{q}_L\gamma^\mu d_L) &= \bar{q}(x)\lambda_{j2}\gamma^\mu\frac{1-\gamma_5}{2}q(x) \equiv L^\mu_{\bar{q}d}(x) = \frac{\delta\mathcal{L}_{\rm QCD}(x)}{\delta l_\mu(x)_{j2}}\,,
\end{aligned}
\tag{3.5}
$$

the bosonization of these left-current densities, to lowest order in the chiral expansion starts at $\mathcal{O}(p)$ and are given by the expressions:

$$L^{\mu}_{\bar{s}q}(x) \Rightarrow \frac{i}{2}F_0^2 \,\mathrm{tr}\left[\lambda_{3j}(D^{\mu}U^{\dagger})U\right] + \mathcal{O}(p^3) \tag{3.6}$$

$$L^{\mu}_{\bar{q}d}(x) \Rightarrow \frac{i}{2}F_0^2 \,\mathrm{tr}\left[\lambda_{j2}(D^{\mu}U^{\dagger})U\right] + \mathcal{O}(p^3)\,. \tag{3.7}$$

The contribution to the coupling constant $\mathbf{g}_{\underline{8}}$ from the *factorized* patterns of the Q_4 and Q_6 penguin operators can now be readily obtained from the definitions in eqs. (1.3), (1.4) and the results in eqs. (3.3), (3.4) and (3.6), (3.7). One thus obtains the well known result [3]

$$g_{\underline{8}}|^{\text{factorized}}_{Q6,Q4} = C_6(\mu)\left[-16L_5\frac{\langle\bar{\psi}\psi\rangle^2}{F_0^2}\frac{1}{F_0^4}\right] + C_4(\mu)\mathbf{1}\,, \tag{3.8}$$

where we have used the fact that

$$B = \frac{1}{F_0^2}|\langle\bar{\psi}\psi\rangle|\,. \tag{3.9}$$

It is well known that in this approximation, *polychromatic penguins don't fly,* as emphasized in ref. [8],

4. Beyond factorization

Because of the chiral structure of the lowest order electroweak lagrangian in eq. (1.4), we have to expand the Dirac propagators in the *unfactorized patterns* in eqs. (2.3) and (2.4) up to two chiral powers in the external sources. There is, however, a certain arbitrariness in the choice of the external sources to expand. The simplest (and recommended) choice is the one which makes it explicit that all the perturbative QCD (pQCD) short-distance contributions are factored out in the Wilson coefficients $C_i(\mu)$. Since the operators Q_4 and Q_6 transform like $\underline{8}_L \times \underline{1}_R$ operators under chiral rotations, expanding the propagators in right-handed r_μ sources will necessarily bring in Green's functions with extra $R_\mu \equiv \bar{q}\gamma_\mu\frac{1+\gamma_5}{2}q$ currents only, and there is no way then that the product of these $(\underline{1}_L \times \underline{8}_R)$ R_μ operators with the initial $(\underline{8}_L \times \underline{1}_R)$ operator can produce a $\underline{1}_L \times \underline{1}_R$ term, which ensures no mixing with possible pQCD contributions in the long-distance evaluation. Technically, this can be accomplished by an appropriate *"tout à droite"* rotation [4] of the quarks fields so that in the rotated basis the right-handed quark field has absorbed all the Goldstone degrees of freedom and the Dirac operator has as a chiral vector connection the external l_μ field only. One way to implement this is as follows: define Q_L and Q_R quark fields, such that

$$Q_L(x) = \xi_L(x)q_L(x) \qquad \text{and} \qquad Q_R(x) = \xi_R(x)q_R(x)\,, \tag{4.1}$$

[3] See e.g. refs. [8, 17, 23] where earlier references can also be found.

[4] In the french cycling jargon, "tout à droite" means to put the front and rear gears so as to reach maximum speed (the biggest front plateau with the smallest rear socket). This is what we recommend to do here with the Dirac operator!

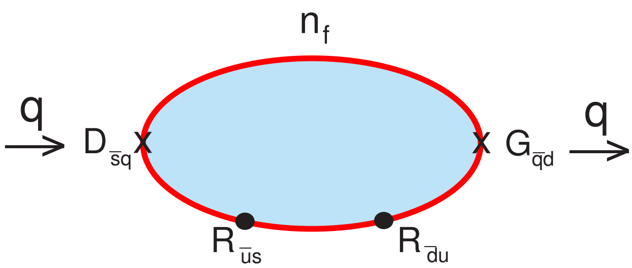

Figure 2: "Eye"-like configurations contributing to the unfactorized bosonization of the Q_6 operator. The solid dots in the quark propagator correspond to external right-handed sources of *soft* quark currents.

where $\xi_L(x)$ and $\xi_R(x)$ are left and right $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R/\,\mathrm{SU}(3)_V$ coset representatives , with $\xi_R(x)\xi_L^\dagger(x) = U(x)$. Then, choose a gauge where

$$\xi_L(x) = 1\,, \qquad \text{while} \qquad \xi_R(x) = U(x)\,. \tag{4.2}$$

The Dirac operator in this *"tout à droite"* rotated basis is rather simple

$$D_\chi = \gamma^\mu\left(\partial_\mu + iG_\mu - il_\mu\frac{1-\gamma_5}{2}\right) + U^\dagger D_\mu U\gamma^\mu\frac{1+\gamma_5}{2} + iU^\dagger\mathcal{M}\,\frac{1-\gamma_5}{2} + i\mathcal{M}^\dagger U\frac{1+\gamma_5}{2}\,. \tag{4.3}$$

The chiral expansion, in the chiral limit where $\mathcal{M} = \mathcal{M}^\dagger = 0$, can then be made in powers of the external source which collects the full Goldstone structure; i.e., the term $U^\dagger D_\mu U\gamma^\mu\frac{1+\gamma_5}{2}$ in eq. (4.3).

4.1 Determination of the unfactorized Green functions

As already mentioned, we are only considering in this work the bosonization of the *unfactorized* terms in eqs. (2.3) and (2.4) of $\mathcal{O}(N_c^2\frac{n_f}{N_c})$ in the $1/N_c$ expansion, and in the chiral limit. As we shall see, the Physics features which already emerge at that level deserve attention. It is possible, however, to extend the calculation in the chiral limit to the rest of the $\mathcal{O}(N_c^2 1/N_c)$ contributions, which are not enhanced by a n_f factor, something which we plan to do in the near future.

Let us discuss the case of the Q_6 operator in some detail. The only configurations which can contribute in the approximation that we are considering, are the so called "eye"-like configurations [26], where the two right-handed sources come out from the same propagator in eq. (2.4), as illustrated in figure 2. They produce the functional integral[5]

$$\begin{aligned}\langle Q_6\rangle|_{\text{eye.}} = 8\;n_f\int\frac{d^4q}{(2\pi)^4}\int d^4y\;\mathrm{Tr}(x|\frac{1+\gamma_5}{2}\frac{i}{\not{p}+\frac{\not{q}}{2}}|y)\;\times\\ \times(y|\lambda_{32}\frac{1-\gamma_5}{2}\frac{i}{\not{p}-\frac{\not{q}}{2}}\left(U^\dagger D_\alpha U\gamma^\alpha\frac{1+\gamma_5}{2}\right)\frac{i}{\not{p}-\frac{\not{q}}{2}}\left(U^\dagger D_\beta U\gamma^\beta\frac{1+\gamma_5}{2}\right)\frac{i}{\not{p}-\frac{\not{q}}{2}}|x),\end{aligned}$$

[5] The contribution from the term where the first propagator in eq. (2.4) is expanded instead of the second one, is modulated by the flavour factor $\sum_j \lambda_{j2}\lambda_{3j} = 0$, and therefore it vanishes.

which gives a term with the required chiral structure: $8n_f \times \mathrm{tr}\,\lambda_{32} U^\dagger D_\alpha U U^\dagger D_\beta U = -8n_f \times \mathrm{tr}\,\lambda_{32} D_\alpha U^\dagger D_\beta U$, modulated by the integral (recall eq. (2.6))

$$\int \frac{d^4q}{(2\pi)^4} \int d^4y\, \mathrm{Tr}(x|\frac{1+\gamma_5}{2}\frac{i}{\not p + \frac{\not q}{2}}|y)(y|\frac{1-\gamma_5}{2}\frac{i}{\not p - \frac{\not q}{2}}\gamma^\alpha \frac{1+\gamma_5}{2}\frac{i}{\not p - \frac{\not q}{2}}\gamma^\beta \frac{1+\gamma_5}{2}\frac{i}{\not p - \frac{\not q}{2}}|x). \tag{4.4}$$

The integral in question can be related to a well defined QCD Green's function, which is the connected Green's function

$$\mathcal{W}^{\alpha\beta}_{\mathrm{DGRR}}(q,k) = \lim_{k\to 0} i^3 \int d^4x d^4y d^4z e^{iq\cdot x} e^{ik\cdot y} e^{-ik\cdot z} \langle 0|T\{D_{\bar{s}q}(x) G_{\bar{q}d}(0) R^\alpha_{\bar{d}u}(y) R^\beta_{\bar{u}s}(z)\}|0\rangle|_{\mathrm{conn.}}\,, \tag{4.5}$$

where (no summation over the flavour index q in eq. (4.5))

$$D_{\bar{s}q}(x) = \bar{q}(x)\lambda_{3j}\frac{1+\gamma_5}{2}q(x)\,, \qquad G_{\bar{q}d}(0) = \bar{q}(0)\lambda_{j2}\frac{1-\gamma_5}{2}q(0)\,, \tag{4.6}$$

and

$$R^\alpha_{\bar{d}u}(y) = \bar{q}(y)\lambda_{21}\gamma^\alpha\frac{1+\gamma_5}{2}q(y)\,, \qquad R^\beta_{\bar{u}s}(z) = \bar{q}(z)\lambda_{13}\gamma^\beta\frac{1+\gamma_5}{2}q(z)\,. \tag{4.7}$$

This Green's function can be viewed as a two-point function of the two $D_{\bar{s}q}(x)$ and $G_{\bar{q}d}(0)$ operators which define Q_6, with two soft insertions of the composite operators $R^\alpha_{\bar{d}u}(y)$ and $R^\beta_{\bar{u}s}(z)$. The relation to the integral in eq. (4.4), and hence to $\langle Q_6\rangle|_{\mathrm{eye.}}$, is the following

$$\langle Q_6\rangle|_{\mathrm{eye.}} = -\,\mathrm{tr}\,\lambda_{32} D_\alpha U^\dagger D_\beta U \times 8n_f \times \left(\int \frac{d^4q}{(2\pi)^4}\mathcal{W}^{\alpha\beta}_{\mathrm{DGRR}}(q,k)\right)\Bigg|_{g^{\alpha\beta}}\,, \tag{4.8}$$

with the integral restricted to the term proportional to $g^{\alpha\beta}$. In fact, this term appears naturally, when performing the integral over the solid angle $d\Omega_q$, which has the form ($Q^2 \equiv -q^2$):

$$\int d\Omega_q\, \mathcal{W}^{\alpha\beta}_{\mathrm{DGRR}}(q,k) = \left(\frac{k^\alpha k^\beta}{k^2} - g^{\alpha\beta}\right)\, \mathcal{W}_{\mathrm{DGRR}}(Q^2)\,, \tag{4.9}$$

where the transversality in the four-vector k follows from Current Algebra Ward identities. We are still left with an integral of the invariant function $\mathcal{W}_{\mathrm{DGRR}}(Q^2)$ over the full euclidean range ($0 \le Q^2 \le \infty$) which has to be done in the same *renormalization scheme* which has been adopted when doing the calculation of the short-distance Wilson coefficient $C_6(\mu)$ in eq. (1.3), i.e. in the $\overline{\mathrm{MS}}$ scheme.

The case of the unfactorized bosonization of the Q_4 operator is rather similar. To $\mathcal{O}(p^2)$ it is governed by a connected Green's function which can be viewed as a two-point function of the two $L^\mu_{\bar{s}q}(x)$ and $L^\nu_{\bar{q}d}(0)$ operators which define Q_4, with two soft insertions of the composite operators $R^\alpha_{\bar{d}u}(y)$ and $R^\beta_{\bar{u}s}(z)$; i.e.,

$$\langle Q_4\rangle|_{\mathrm{eye.}} = \mathrm{tr}\,\lambda_{32} D_\alpha U^\dagger D_\beta U \times 4n_f \times \left(g_{\mu\nu}\int \frac{d^4q}{(2\pi)^4}\mathcal{W}^{\mu\nu\alpha\beta}_{\mathrm{LLRR}}(q,k)\right)\Bigg|_{g^{\alpha\beta}}\,, \tag{4.10}$$

where here

$$\mathcal{W}^{\mu\nu\alpha\beta}_{\mathrm{LLRR}}(q,k) = \lim_{k\to 0} i^3 \int d^4x d^4y d^4z e^{iq\cdot x} e^{ik\cdot y} e^{-ik\cdot z} \langle 0|T\{L^\mu_{\bar{s}q}(x) L^\nu_{\bar{q}d}(0) R^\alpha_{\bar{d}u}(y) R^\beta_{\bar{u}s}(z)\}|0\rangle|_{\mathrm{conn.}}\,, \tag{4.11}$$

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with no summation over the flavour q index in the currents. The Ward identities again ensure that the integral over the solid angle in eq. (4.10) also depends on one invariant function only,

$$g_{\mu\nu}\int d\Omega_q\;\mathcal{W}^{\mu\,\nu\,\alpha\,\beta}_{\rm LLRR}(q,k)=\left(\frac{k^\alpha k^\beta}{k^2}-g^{\alpha\beta}\right)\;\mathcal{W}_{\rm LLRR}(Q^2)\,, \tag{4.12}$$

and the term proportional to $g^{\alpha\beta}$ appears then naturally, as well.

It is now possible to express the contribution to the coupling constant $g_{\underline{8}}$ of the electroweak chiral lagrangian in eq. (1.4) from the *factorized patterns* of the Q_4 and Q_6 penguin operators to $\mathcal{O}(N_c^2)$, as well as from their *unfactorized patterns* of $\mathcal{O}(N_c^2\frac{n_f}{N_c})$, which appear as integrals of the invariant functions $\mathcal{W}_{\rm DGRR}(Q^2)$ and $\mathcal{W}_{\rm LLRR}(Q^2)$, in the following way:

$$\mathbf{g}_{\underline{8}}|_{Q_4,Q_6}=C_6(\mu)\left\{-16L_5\frac{\langle\bar\psi\psi\rangle^2}{F_0^6}+\frac{8n_f}{16\pi^2F_0^4}\left[\frac{(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)}\int_0^\infty dQ^2(Q^2)^{1-\epsilon/2}\mathcal{W}_{\rm DGRR}(Q^2)\right]_{\overline{\rm MS}}\right\}+$$
$$+C_4(\mu)\left\{1-\frac{4n_f}{16\pi^2F_0^4}\left[\frac{(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)}\int_0^\infty dQ^2(Q^2)^{1-\epsilon/2}\mathcal{W}_{\rm LLRR}(Q^2)\right]_{\overline{\rm MS}}\right\}\,. \tag{4.13}$$

Notice that the integrals over Q^2 are $\overline{\rm MS}$ renormalized; i.e., poles in $1/\epsilon$ as well as the constant $\log 4\pi-\gamma_E$ are removed. Let us remember, however, that the scale dependence of the renormalized constant $L_5(\mu)$ in χPT is

$$L_5(\mu)=L_5(M_\rho)-\frac{\Gamma_5}{16\pi^2}\log\frac{\mu}{M_\rho}\,,\qquad\text{with}\qquad\Gamma_5=\frac{n_f}{8}\,, \tag{4.14}$$

and coupling constants in χPT have been renormalized in the Bern $\overline{\rm MS}$ scheme where poles in $1/\epsilon$, as well as the constant $\log 4\pi-\gamma_E+1$, are removed. This difference of $\overline{\rm MS}$ renormalization schemes has to be taken into account consistently in order to exhibit the overall μ scale cancellation between the short- and long-distance contributions, at the order of approximation that we are working in the $1/N_c$ expansion. Here we have chosen to renormalize all the UV divergences with a single scale μ^ϵ. Therefore, in eq. (4.13) there is a μ-scale dependence both in parameters such as $L_5(\mu)$ and $\langle\bar\psi\psi\rangle(\mu)$, as well as in the short-distance Wilson coefficients, such as $C_{6,4}(\mu)$. There is, however, freedom to renormalize L_5 with a different scale, which could be called $\nu_{\rm chiral}$ as conventionally done in χPT. That this is possible is due to the fact that the renormalization of L_5 does not mix with that of the quark-condensate, or the Wilson coefficient. Notice also that, in the $\overline{\rm MS}$ renormalization scheme which we are using, where quark masses (and hence the pseudoscalar masses) are set to zero, there are no chiral loop contributions to the factorized term in eq. (4.13). The fact that all the UV scale μ dependence cancels out in the end in the coupling constant $g_{\underline{8}}$ is a nontrivial check that the interplay between short- and long-distances has been brought under control.

4.2 Why one could expect large unfactorized contributions

At this level, we think it necessary to have a short discussion explaining the relative importance of the *factorized* versus *unfactorized* contributions in eq. (4.13). It has long been recognized that the calculation of the coupling constant $\mathbf{g}_{\underline{8}}$ is a multi-scale problem. Already at short distances, the perturbative running of the Wilson coefficients has to deal with several different scales from M_W down to the charm mass. However, it is perhaps not so much emphasized that, even in the long-distance regime, there are also two distinctive scales playing a role in eq. (4.13). Although in QCD all scales are ultimately related to $\Lambda_{\overline{\rm MS}}$, it is a fact of life that F_0 is much smaller than a typical hadronic scale, of the size of an average resonance mass, $M_R \sim 1\,\mathrm{GeV}$. Also the typical scale defined by a QCD vacuum condensate,[6] Λ_c, is known to be smaller than M_R. One then has that, roughly, $F_0 \ll \Lambda_c \ll M_R$. Because F_0^2 is of $\mathcal{O}(N_c)$ while M_R^2 is of $\mathcal{O}(1)$ a naive interpretation of the large-N_c counting could lead one to conclude that $F_0^2 \gg M_R^2$, which is obviously wrong. It is this hierarchy of scales which is at the origin of why the *unfactorized* contribution, although of $\mathcal{O}(N_c)$, may be as large as — or sometimes larger than — the *factorized* one, even though the latter is formally of $\mathcal{O}(N_c^2)$. For instance, we already found that in the case of the calculation of B_K in the chiral limit [12, 16], the unfactorized contributions are of the order of 50%. In the present case of the $Q_6\,, Q_4$ penguin operators, the unfactorized contributions actually turn out to be much larger than the factorized ones, as we shall soon see. We conclude that in the case of four-quark operators, a naive interpretation of the $1/N_c$ expansion may be misleading and has to be done *after* the contributions from the different physical scales have been separated.

In the case we are considering, knowing that (as will be confirmed in the next section by explicit calculation):

$$\int_0^\infty dQ^2 Q^2\, \mathcal{W}_{\rm DGRR}(Q^2) \sim B^2 F_0^2\,, \qquad \text{and} \qquad \int_0^\infty dQ^2 Q^2\, \mathcal{W}_{\rm LLRR}(Q^2) \sim F_0^2 M_R^2\,, \tag{4.15}$$

up to logarithmic factors, and that $L_5 \sim F_0^2/M_R^2$, the ratio of the *unfactorized* to the *factorized* contributions in eq. (4.13) is given by precisely the ratio of scales

$$\frac{\text{unfactorized}}{\text{factorized}} \sim \frac{M_R^2}{16\pi^2 F_0^2}\,, \tag{4.16}$$

which is a number of $\mathcal{O}(1)$, although large-N_c suppressed. This is true for *both* the contributions from Q_6 and Q_4 to $g_{\underline{8}}$ in eq. (4.13).

Is this a shortcoming of the large$-N_c$ expansion? Not necessarily. It is rather that the standard analysis, as used, e.g., in ref. [25] was only applied to situations in which the two scales F_0 and M_R do not compete for the control of the large-N_c expansion, as it is the case in eq. (4.13). In fact, we think that there is nothing specially new about this dynamical effect of scales. Even the perturbative expansion in QED can be affected by the presence of two widely separated masses, because they give rise to large logarithms of their ratio. In the case of the electroweak theory, Bardeen *et al.* in ref. [17] were the first to point out that the large-N_c expansion is also affected by the presence of the large value of $\log M_W/m_{\rm charm}$. The point that we would like to emphasize here is that, also in the low-energy region, ratios of scales such as $M_R^2/(4\pi F_0)^2$ may upset a large-N_c counting

[6] We are thinking here of something like $\langle\bar\psi\psi\rangle^{1/3}$ at $\mu = 1\,\mathrm{GeV}$.

done in a naive way. On the contrary, once the contributions from these two scales have been clearly separated, it should be safer applying a large-N_c argument. For instance, one might fear that since the unfactorized term is larger than the factorized one, the whole expansion breaks down and the next subleading terms will be even bigger. Although a definitive answer to this point can only come from the solution to QCD in the large-N_c expansion, which is not known, we would like to argue that there is no reason to expect this breakdown. Subleading terms to those of eq. (4.13) are expected to give contributions of the order of the width of the resonances involved, or of the violation of the Zweig rule. There is presently no evidence against these effects being reasonably small except, perhaps, in some very specific cases where the effect of the strong Goldstone-Goldstone interaction in the $J = I = 0$ channel appears, which may deserve special attention and which we plan to study elsewhere.

4.3 Long and short distance constraints on the functions $\mathcal{W}_{\rm DGRR}(Q^2)$ and $\mathcal{W}_{\rm LLRR}(Q^2)$

The low-Q^2 behaviour of these functions is governed by χPT. An explicit calculation gives the results:

$$\lim_{Q^2\to 0}\mathcal{W}_{\rm DGRR}(Q^2) = \frac{1}{8}\times\frac{BF_0}{Q^2}\times\frac{BF_0}{Q^2}+\left(-L_5+\frac{5}{2}L_3\right)\frac{B^2}{Q^2}+\mathcal{O}(\mathrm{Cte.})\,, \tag{4.17}$$

$$\lim_{Q^2\to 0}\mathcal{W}_{\rm LLRR}(Q^2) = -\frac{3}{8}\frac{F_0^2}{Q^2}+\left(-\frac{15}{2}L_3+\frac{3}{2}L_9\right)+\mathcal{O}(Q^2)\,. \tag{4.18}$$

The double pole contribution to $\mathcal{W}_{\rm DGRR}(Q^2)$, the L_5 contribution to $\mathcal{W}_{\rm DGRR}(Q^2)$ and the simple pole contribution to $\mathcal{W}_{\rm LLRR}(Q^2)$ agree with the result first obtained in refs. [18, 20] and [19] respectively.[7] To our knowledge, the other terms have not been calculated before.

On the other hand, the high-Q^2 behaviour of the functions $\mathcal{W}_{\rm DGRR}(Q^2)$ and $\mathcal{W}_{\rm LLRR}(Q^2)$ is governed by the operator product expansion of the $D\bigotimes G$ density currents in eq. (4.5) and the $L\bigotimes L$ currents in eq. (4.11), with the results:

$$\lim_{Q^2\to\infty}\mathcal{W}_{\rm DGRR}(Q^2) = -\frac{1}{6}\,\pi^2\frac{\alpha_{\rm s}}{\pi}\frac{F_0^4}{Q^4}+\frac{8}{3}\,\pi^2\frac{\alpha_{\rm s}}{\pi}\frac{\langle\bar\psi\psi\rangle^2}{F_0^2}\frac{L_5}{Q^4}+\mathcal{O}\left[\epsilon\frac{1}{Q^4}\right], \tag{4.19}$$

$$\lim_{Q^2\to\infty}\mathcal{W}_{\rm LLRR}(Q^2) = +\frac{1}{3}\,\pi^2\frac{\alpha_{\rm s}}{\pi}\frac{F_0^4}{Q^4}-\frac{16}{3}\pi^2\frac{\alpha_{\rm s}}{\pi}\frac{\langle\bar\psi\psi\rangle^2}{F_0^2}\frac{L_5}{Q^4}+\mathcal{O}\left[\epsilon\frac{1}{Q^4}\right]. \tag{4.20}$$

We remark that in eqs. (4.17)–(4.20) the chiral couplings $L_{5,9}$ are the ones corresponding to leading order at large-N_c and, therefore, do not run with the scale.

In this work we shall content ourselves with only the leading-log approximation. This allows us to do away with the evaluation of the terms of $\mathcal{O}[\epsilon\frac{1}{Q^4}]$ in the OPE, which means that we shall not be able to specify *scheme* dependences in our $\overline{\rm MS}$-renormalization calculation of the long-distance effects.

[7]The L_3 contribution to eq. (4.17) was omitted in ref. [20] for reasons which turn out to be incorrect.

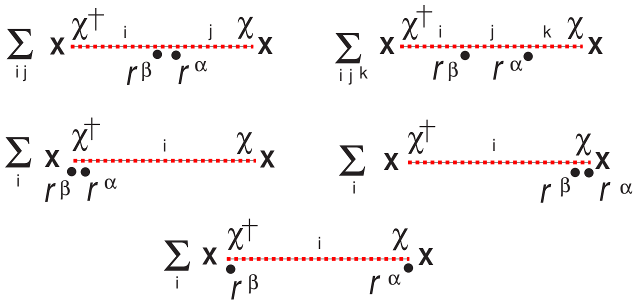

Figure 3: Different types of hadronic tree diagrams which can contribute to the Green's functions $Q^2\mathcal{W}_{\rm DGRR}(Q^2)$ and $Q^2\mathcal{W}_{\rm LLRR}(Q^2)$ (replacing $\chi \to l$) in the large-N_c limit. The indices i, j, k label the possible poles.

In large-N_c QCD, the Green's functions $\mathcal{W}_{\rm DGRR}(Q^2)$ and $\mathcal{W}_{\rm LLRR}(Q^2)$ are meromorphic functions in the Q^2 variable. The most general structure they can have is:

$$Q^2\mathcal{W}_{\rm DGRR}(Q^2) = \frac{1}{8}\frac{(BF_0)^2}{Q^2} + \tag{4.21}$$
$$+\mu_{\rm had.}^4 \sum_i \frac{\alpha_i}{Q^2+M_i^2} + \mu_{\rm had.}^6 \sum_i \frac{\beta_i}{(Q^2+M_i^2)^2} + \mu_{\rm had.}^8 \sum_i \frac{\gamma_i}{(Q^2+M_i^2)^3}\,,$$

$$Q^2\mathcal{W}_{\rm LLRR}(Q^2) = \mu_{\rm had.}^4 \sum_j \frac{\alpha'_j}{Q^2+M_j^2} + \mu_{\rm had.}^6 \sum_i \frac{\beta'_j}{(Q^2+M_j^2)^2} + \mu_{\rm had.}^8 \sum_i \frac{\gamma'_j}{(Q^2+M_j^2)^3}\,, \tag{4.22}$$

where $\mu_{\rm had.}$ is an arbitrary hadronic mass scale, so as to make the residues α_i, β_i, γ_i and α'_j, β'_j, γ'_j dimensionless. This scale can be interpreted as the scale at which the four-quark lagrangian in eq. (1.3) is matched onto the chiral lagrangian of eq. (1.4), i.e. the scale at which the meson resonances are integrated out. Physical results, however, do not depend on the particular choice of the scale $\mu_{\rm had.}$, except for higher order terms. The sums are, in principle, extended to an infinite number of states, as illustrated in figure 3. In practice, it is useful to start with the *minimal hadronic approximation*[8] (MHA) where the number of non-Goldstone states is limited to a minimum number of lowest mass states, with appropriate quantum numbers; the minimum number which is necessary to satisfy the leading OPE constraints. In our case, the MHA we shall adopt is the one with a 1^- vector pole of mass M_V and a 0^+ scalar pole of mass M_S.

With the notation ($M_V = 770\,\mathrm{MeV}$ and $M_S \simeq 1\,\mathrm{GeV}$)

$$z = \frac{Q^2}{\mu_{\rm had.}^2}\,, \quad \rho_V = \frac{M_V^2}{\mu_{\rm had.}^2} \quad \text{and} \quad \rho_S = \frac{M_S^2}{\mu_{\rm had.}^2}\,, \tag{4.23}$$

the MHA parameterizations are,

$$z\mathcal{W}_{\rm DGRR}^{(\rm MHA)}(z\mu_{\rm had.}^2) = \frac{1}{8}\left(\frac{BF_0}{\mu_{\rm had.}^2}\right)^2 \frac{1}{z} + \frac{\alpha_V}{z+\rho_V} + \frac{\alpha_S}{z+\rho_S}\,, \tag{4.24}$$

[8]See e.g. ref. [10] and references therein for details

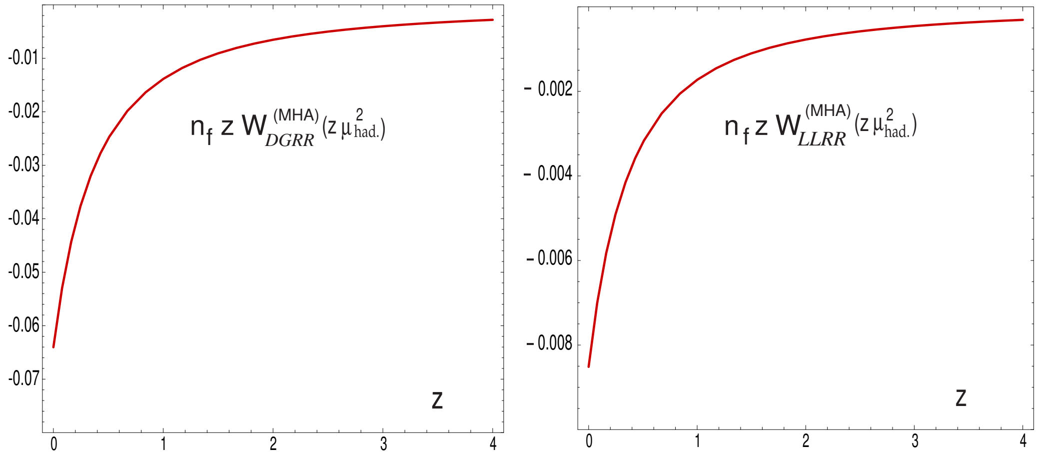

Figure 4: The functions $z\mathcal{W}^{(\mathrm{MHA})}_{\mathrm{DGRR}}(z\mu^2_{\mathrm{had.}})$ (with the Goldstone pole removed) and $z\mathcal{W}^{(\mathrm{MHA})}_{\mathrm{LLRR}}(z\mu^2_{\mathrm{had.}})$ in eqs. (4.24) and (4.25), versus $z = Q^2/\mu^2_{\mathrm{had.}}$ for the particular choice $\mu_{\mathrm{had.}} = 1\,\mathrm{GeV}$ and $|\langle\bar{\psi}\psi\rangle|^{1/3} = 250\,\mathrm{MeV}$. The values of the other input parameters are specified in the appendix. Notice also the different vertical scales in the two plots.

and

$$z\mathcal{W}^{(\mathrm{MHA})}_{\mathrm{LLRR}}(z\mu^2_{\mathrm{had.}}) = \frac{\alpha'_V}{z+\rho_V} + \frac{\alpha'_S}{z+\rho_S}\,, \tag{4.25}$$

with the residues α_V, α_S, α'_V and α'_S solutions of the system of equations defined by the short-distance constraints

$$\frac{1}{8}(BF_0)^2 + \mu^4_{\mathrm{had.}} \sum_{i=V,S} \alpha_i = -\frac{1}{6}\,\pi^2\frac{\alpha_{\mathrm{s}}}{\pi}F_0^4 + \frac{8}{3}\,\pi^2\frac{\alpha_{\mathrm{s}}}{\pi}\frac{\langle\bar{\psi}\psi\rangle^2}{F_0^2}L_5\,, \tag{4.26}$$

$$\mu^4_{\mathrm{had.}} \sum_{j=V,S} \alpha'_j = +\frac{1}{3}\,\pi^2\frac{\alpha_{\mathrm{s}}}{\pi}F_0^4 - \frac{16}{3}\pi^2\frac{\alpha_{\mathrm{s}}}{\pi}\frac{\langle\bar{\psi}\psi\rangle^2}{F_0^2}L_5\,, \tag{4.27}$$

and the long-distance constraints

$$\mu^2_{\mathrm{had.}} \sum_{i=V,S} \frac{1}{\rho_i}\,\alpha_i = -\left(L_5 - \frac{5}{2}L_3\right)B^2\,, \tag{4.28}$$

$$\mu^2_{\mathrm{had.}} \sum_{j=V,S} \frac{1}{\rho_j}\alpha'_j = -\frac{3}{8}F_0^2\,. \tag{4.29}$$

The shapes of the resulting functions in eqs. (4.24) and (4.25) are shown in figure 4. A few comments are now in order:

- The reason why we are showing the shape of the functions $zW^{(\mathrm{MHA})}_{\mathrm{DGRR}}(z)$ and $zW^{(\mathrm{MHA})}_{\mathrm{LLRR}}(z)$, instead of $W^{(\mathrm{MHA})}_{\mathrm{DGRR}}(z)$ and $W^{(\mathrm{MHA})}_{\mathrm{LLRR}}(z)$ themselves, is that these are precisely the relevant integrands which appear in the integrals in eq. (4.13). The pion pole in $z \times W^{(\mathrm{MHA})}_{\mathrm{DGRR}}(z)$ does not contribute to the "area" in dimensional regularization, and this is why we can remove it from the integrand with impunity.

- In spite of the fact that the Goldstone pole (i.e. the term proportional to $(BF_0)^2$ in eq. (4.24)) does not contribute to the Q^2 integral in dimensional regularization, the sum of the residues of the hadronic simple poles $\sum_i \alpha_i$ is constrained by the *residue* of this Goldstone pole through the OPE, as shown in eq. (4.26); and in fact, it is this term which largely dominates the short-distance constraint. This is the subtle way in which the lowest order chiral lagrangian, which does not contribute at the factorized level, shows its presence at the unfactorized level. It is precisely this term which is responsible for the renormalization of L_5.

- The residue of the single Goldstone pole contribution to $\mathcal{W}_{\rm DGRR}$ is fixed by a combination of couplings of the $\mathcal{O}(p^4)$ chiral lagrangian, as seen in eq. (4.17). The same residue provides a constraint on the sum $\sum_i \frac{\alpha_i}{\rho_i}$ of the hadronic parameters, as seen in eq. (4.28).

- Numerically, the chiral factor $L_5 - \frac{5}{2}L_3 \simeq 10^{-2}$ which appears in eqs. (4.17) and (4.28) is rather large, and it is at the origin of the fact that the *unfactorized* contributions will turn out to be so important, since it is this quantity (times $\langle\bar{\psi}\psi\rangle^2$) which fixes the value at the origin of the integrand $Q^2 \times W_{\rm DGRR}(Q^2)$ in eq. (4.13).

- The corresponding chiral slope factor $-\frac{15}{2}L_3 + \frac{3}{2}L_9 \simeq 30 \times 10^{-3}$ for $\mathcal{W}_{\rm LLRR}(Q^2)$ in eq. (4.18) is even larger. However, in this case, the intercept at the origin is fixed by the $\mathcal{O}(p^2)$ term $-\frac{3}{8}F_0^2$ (see eq. (4.29)), which is of a reasonable size.

The MHA parameterizations of the functions $\mathcal{W}^{(\rm MHA)}_{\rm DGRR}(Q^2)$ and $\mathcal{W}^{(\rm MHA)}_{\rm LLRR}(Q^2)$ may, however, be improved. On the short-distance side, they only approach their OPE behaviour at rather large Q^2 values. On the long-distance side the behaviour of $\mathcal{W}^{(\rm MHA)}_{\rm LLRR}(Q^2)$ at small Q^2 fails to reproduce the known slope in eq. (4.18). This is why we also went beyond the MHA and considered more sophisticated parameterizations of the general structure given in eqs. (4.21) and (4.22), demanding that they interpolate smoothly between the known chiral behaviour and the OPE behaviour. In the case of $\mathcal{W}_{\rm DGRR}(Q^2)$ we included a vector simple pole, a scalar simple and double poles, and an excited pseudoscalar pole. For $\mathcal{W}_{\rm LLRR}(Q^2)$ we included a scalar simple pole together with simple, double and triple pole for a vector, an excited vector, and an axial-vector states. The extra information to fix the residues of the poles was obtained by demanding that $\mathcal{W}_{\rm DGRR}(Q^2)$ and $\mathcal{W}_{\rm LLRR}(Q^2)$ reproduce the OPE at various points, $Q^2 \gtrsim 9\,\mathrm{GeV}^2$. This scale was arbitrarily chosen as being low enough to help producing a smooth interpolation but high enough for the OPE behaviour to be trusted. Typical examples of the shapes we get for $n_f z\mathcal{W}_{\rm DGRR}(Q^2)$ and $n_f z\mathcal{W}_{\rm LLRR}(Q^2)$ are shown, respectively, in figures 5 and 6 below (the thick solid curves). For the purpose of comparison, we also show in the same plots the OPE behaviour (the thick dashed curves), as well as the χPT slope in the case of $z\mathcal{W}_{\rm LLRR}(Q^2)$ (the thin dashed line in figure 6). The slope of $z\mathcal{W}_{\rm DGRR}(Q^2)$ (with the pion pole removed) could be determined from the couplings of the $\mathcal{O}(p^6)$ chiral lagrangian. The calculation, which is rather involved, is beyond the scope of this paper.

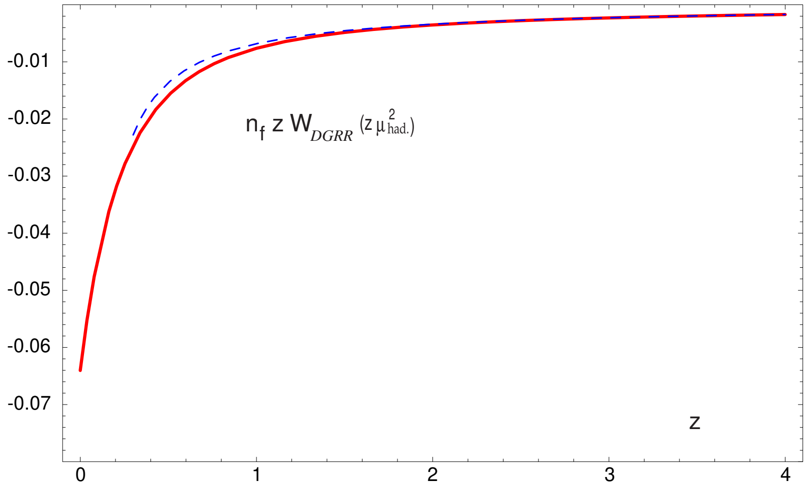

Figure 5: Shape of $z\mathcal{W}_{\rm DGRR}(z\mu^2_{\rm had.})$ (pion pole removed) versus z (solid curve) for a large-N_c type parameterization as in eq. (4.21) which matches *smoothly* the short-distance OPE behaviour (the thick dashed line) with the long-distance χPT behaviour. The curves correspond to $\mu_{\rm had.} = 1\,\text{GeV}$, $L_5|_{\text{large-}N_c} = 10^{-3}$ and $|\langle\bar{\psi}\psi\rangle|^{1/3} = 250\,\text{MeV}$; the values of the other input parameters are specified in the appendix A.

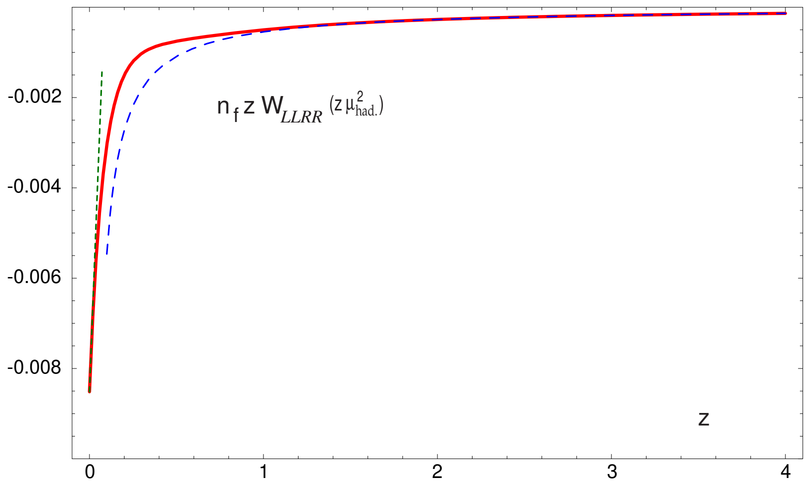

Figure 6: Shape of $z\mathcal{W}_{\rm LLRR}(z\mu^2_{\rm had.})$ versus z (solid curve) for a large-N_c type parameterization as in eq. (4.22) which matches *smoothly* the short-distance OPE behaviour (the thick dashed line) with the long-distance χPT behaviour i.e., the point at the origin and the slope (the thin dashed line). The curves correspond to $\mu_{\rm had.} = 1\,\text{GeV}$, $L_5|_{\text{large-}N_c} = 10^{-3}$ and $|\langle\bar{\psi}\psi\rangle|^{1/3} = 250\,\text{MeV}$; the values of the other input parameters are specified in the appendix A.

5. Analytical results for the coupling constant $\underline{\mathbf{g}}_8$ from QCD penguins

We are now in a position to do the integrals in eq. (4.13) and hence to obtain an evaluation of the contribution to $\underline{\mathbf{g}}_8$ from QCD penguins, beyond the factorization result in eq. (3.8). We shall keep the two-loop evaluation of the Wilson coefficients C_4 and C_6, as obtained e.g. in ref. [33], down to the charm mass scale $m_c = 1.3\,\text{GeV}$; while the evolution from m_c

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to an arbitrary hadronic scale $\mu_{\rm had.} < m_c$ is done at the leading log approximation, which is consistent with the bosonization of the Q_4 and Q_6 operators we have done. This results in the following expression for $\mathbf{g}_{\underline{8}}|_{Q_6,Q_4}$ valid to leading and next-to-leading order in the $1/N_c$ expansion, including terms of $\mathcal{O}(\frac{n_f}{N_c})$:

$$\mathbf{g}_{\underline{8}}|_{Q_6,Q_4} = C_6(m_c) f_6(m_c\,;\mu_{\rm had.}) + C_4(m_c) f_4(m_c\,;\mu_{\rm had.})\,, \tag{5.1}$$

where

$$\begin{aligned} f_6(m_c\,;\mu_{\rm had.}) =& \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{-\frac{9}{11-2\frac{n_f}{N_c}}} \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{\frac{1}{11}\frac{n_f}{N_c}} \times \\ &\times \left[-16 L_5(\mu_{\rm had.}) \frac{\langle\bar\psi\psi\rangle^2_{\mu_{\rm had.}}}{F_0^6} - 8 n_f \frac{\mu_{\rm had.}^4}{16\pi^2 F_0^4} \sum_i \left(\alpha_i \log\rho_i - \frac{\beta_i}{\rho_i} - \frac{1}{2}\frac{\gamma_i}{\rho_i^2}\right)\right] + \\ &+ \frac{1}{9}\frac{n_f}{N_c}\left[1 - \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{-9/11}\right]\,, \end{aligned} \tag{5.2}$$

and

$$\begin{aligned} f_4(m_c\,;\mu_{\rm had.}) =& \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{\frac{1}{11}\frac{n_f}{N_c}} \times \left[1 + 4 n_f \frac{\mu_{\rm had.}^4}{16\pi^2 F_0^4} \sum_j \left(\alpha_j' \log\rho_j - \frac{\beta_j'}{\rho_j} - \frac{1}{2}\frac{\gamma_j'}{\rho_j^2}\right)\right] + \\ &+ \frac{1}{9}\frac{n_f}{N_c}\left[1 - \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{-9/11}\right] \times \left(-16 L_5 \frac{\langle\bar\psi\psi\rangle^2_{\mu_{\rm had.}}}{F_0^6}\right)\,. \end{aligned} \tag{5.3}$$

It is worthwhile to compare the functions $f_6(m_c;\mu_{\rm had.})$ and $f_4(m_c;\mu_{\rm had.})$ with the corresponding expressions obtained from *factorization*; i.e.,

$$\begin{aligned} f_6(m_c\,;\mu_{\rm had.})|_{\rm factorized} &= \left(\frac{\alpha_{\rm s}(m_c)}{\alpha_{\rm s}(\mu_{\rm had.})}\right)^{-9/11} \left[-16 L_5(\mu_{\rm had.}) \frac{\langle\bar\psi\psi\rangle^2_{\mu_{\rm had.}}}{F_0^6}\right]\,, \\ f_4(m_c\,;\mu_{\rm had.})|_{\rm factorized} &= 1\,. \end{aligned} \tag{5.4}$$

We observe that $f_6(m_c\,;\mu_{\rm had.})|_{\rm factorized}$ has a rather strong dependence on the choice of the hadronic scale $\mu_{\rm had.}$ (a fact which is often ignored in phenomenological discussions on ϵ'/ϵ in the literature), while $f_6(m_c;\mu_{\rm had.})$ has a remarkably smooth dependence. This behaviour is illustrated in figure 7, where we plot these functions, normalized to their respective values at $\mu_{\rm had.} = 1\,\mathrm{GeV}$; i.e.,

$$\tilde f_6(m_c\,;\mu_{\rm had.}) \equiv \frac{f_6(m_c\,;\mu_{\rm had.})}{f_6(m_c;1\ \mathrm{GeV})} \qquad \text{and} \qquad \tilde f_6(m_c;\mu_{\rm had.})_{\rm factorized} \equiv \frac{f_6(m_c;\mu_{\rm had.})_{\rm factorized}}{f_6(m_c\,;1\ \mathrm{GeV})_{\rm factorized}} \tag{5.5}$$

versus $\mu_{\rm had.}$ in the range $0.8\,\mathrm{GeV} \le \mu_{\rm had.} \le 1.3\,\mathrm{GeV}$. The reason why we normalize the f_6 functions to their value at $\mu_{\rm had.}=1\,\mathrm{GeV}$ is that, in absolute value, $f_6(m_c\,;\mu_{\rm had.})$ turns out to be larger than $f_6(m_c;\mu_{\rm had.})_{\rm factorized}$; for example, at $\mu_{\rm had.} = 0.8\,\mathrm{GeV}$, we find that

$$f_6(m_c\,;\ 0.8\ \mathrm{GeV}) \sim 3 \times f_6(m_c\,;\ 0.8\ \mathrm{GeV})_{\rm factorized}\,, \tag{5.6}$$

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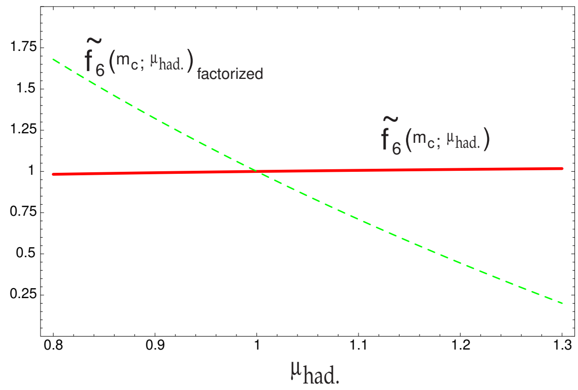

Figure 7: The dependence on the choice of the $\mu_{\rm had.}$ scale of the functions $f_6(m_c;\mu_{\rm had.})$ (the flat solid curve) and $f_6(m_c;\mu_{\rm had.})_{\rm factorized}$ (the dotted curve), normalized to their respective values at $\mu_{\rm had.}=1\,{\rm GeV}$. See the definitions in eq. (5.5).

which is an important enhancement.[9] We also find a similar enhancement, though perhaps less dramatic, of the unfactorized contribution to $f_4(m_c\,;\mu_{\rm had.})$, which as we shall see later, is a welcome feature towards a phenomenological understanding of the observed $\Delta I = 1/2$ rule.

6. Phenomenology of $K \to \pi\pi$ amplitudes and the $\Delta I = 1/2$ rule

We shall define the decomposition of physical $K \to \pi\pi$ amplitudes into isospin amplitudes $A_{I=0,2}$, as follows:

$$A[K^0 \to \pi^+\pi^-] = iA_0 e^{i\delta_0} + \frac{1}{\sqrt{2}} iA_2 e^{i\delta_2}\,, \tag{6.1}$$

$$A[K^0 \to \pi^0\pi^0] = iA_0 e^{i\delta_0} - \sqrt{2} iA_2 e^{i\delta_2}\,, \tag{6.2}$$

$$A[K^+ \to \pi^+\pi^0] = \frac{3}{2} iA_2 e^{i\delta_2}\,, \tag{6.3}$$

where, δ_0 and δ_2 are the $J=0$ $\pi\pi$ phase shifts with $I=0$ and $I=2$ at the K mass. The chiral lagrangian in eq. (1.4) gives the following contributions to the A_I isospin amplitudes

$$\begin{aligned} A_0 &= -\frac{G_{\rm F}}{\sqrt{2}} V_{ud}V_{us}^*\,\sqrt{2}F_0 \left\{ \left(\mathbf{g}_{\underline{8}} + \frac{1}{9}\mathbf{g}_{\underline{27}}\right)(M_K^2 - m_\pi^2) - \frac{2}{3}\frac{1}{F_0^4}\; e^2 \mathbf{g}_{\underline{\rm ew}} \right\}, \\ A_2 &= -\frac{G_{\rm F}}{\sqrt{2}} V_{ud}V_{us}^*\;2F_0 \left\{ \frac{5}{9}\mathbf{g}_{\underline{27}}(M_K^2 - m_\pi^2) - \frac{1}{3}\frac{1}{F_0^4}\; e^2 \mathbf{g}_{\underline{\rm ew}} \right\}. \end{aligned} \tag{6.4}$$

In the large-N_c limit, when only the *factorized* contributions are taken into account, there is a dynamical symmetry which relates the $\mathbf{g}_{\underline{8}}$ and $\mathbf{g}_{\underline{27}}$ couplings:

$$\mathbf{g}_{\underline{8}}|_{\mathcal{O}(N_c^2)} = \mathbf{g}_{\underline{27}}|_{\mathcal{O}(N_c^2)} = \frac{3}{5}\,, \tag{6.5}$$

[9] This is in qualitative agreement with the numerical results found by the authors of refs. [21, 22] within the framework of the extended Nambu-Jona-Lasinio model (ENJL).

while we recall that, experimentally,

$$|\mathbf{g}_{\underline{8}}|_{\text{exp.}} \simeq 5.1 \qquad \text{and} \qquad |\mathbf{g}_{\underline{27}}|_{\text{exp.}} \simeq 0.29 \; : \tag{6.6}$$

although the values to be explained can be reduced to

$$|\mathbf{g}_{\underline{8}}|_{\text{exp.}} \sim 3.3 \qquad \text{and} \qquad |\mathbf{g}_{\underline{27}}|_{\text{exp.}} \sim 0.23 \,, \tag{6.7}$$

if one takes into account the enhancement already provided by the calculated $\mathcal{O}(p^4)$ chiral corrections [28]–[31].

The *unfactorized* $\mathcal{O}(N_c)$ contributions break the dynamical symmetry which is at the origin of the disastrous prediction in eq. (6.5); but, as was pointed out in ref. [32], there still remains a smaller dynamical symmetry at that level of approximation. This symmetry relates the weak matrix elements of the Q_2 and Q_1 four-quark operators in eq. (1.2) (neglecting their mixing with the penguin operators) to those of the $\Delta S = 2$ operator

$$Q_{\Delta S=2} = (\bar{s}_L \gamma^\mu d_L)((\bar{s}_L \gamma_\mu d_L) \,, \quad \langle \bar{K^0} | Q_{\Delta S=2}(0) | K^0 \rangle = f_K^2 M_K^2 \; g_{\Delta S=2}(\mu) \,, \tag{6.8}$$

in the following way:

$$\text{Re}\mathbf{g}_{\underline{8}}|_{Q_2\,,Q_1} = z_1(\mu) \left(-1 + \frac{3}{5} g_{\Delta S=2}(\mu) \right) + z_2(\mu) \left(1 - \frac{2}{5} g_{\Delta S=2}(\mu) \right) , \tag{6.9}$$

and

$$\mathbf{g}_{\underline{27}} = [z_1(\mu) + z_2(\mu)] \, \frac{3}{5} g_{\Delta S=2}(\mu) \,, \tag{6.10}$$

where we are using the same definition of the Wilson coefficients as in Buras *et al.* [33]; i.e.,

$$C_i(\mu) = z_i(\mu) + \tau y_i(\mu) \,, \qquad \text{with} \qquad \tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \,. \tag{6.11}$$

Our present work updates this dynamical symmetry with three new ingredients:

1. The calculation of the constant $g_{\Delta S=2}(\mu)$, within the same framework of the $1/N_c$ expansion discussed here, which has been reported in refs. [12, 16].

2. Our new result in eq. (5.1) adds an extra contribution to the real part of the $\mathbf{g}_{\underline{8}}$ coupling constant

$$\text{Re}\mathbf{g}_{\underline{8}}|_{Q_6\,,Q_4} = z_6(\mu) f_6(\mu\,;\mu_{\text{had.}}) + z_4(\mu) f_4(\mu\,;\mu_{\text{had.}}) \,. \tag{6.12}$$

3. The contribution to $\text{Re}\mathbf{g}_{\underline{8}}$ from the *eye-like* configuration of the Q_2 operator, at $\mathcal{O}(N_c)$, can be read off straightforwardly from our calculation of the hadronization of the Q_4 penguin operator, and adds an important extra contribution

$$\text{Re}\mathbf{g}_{\underline{2}}|_{Q_2\,,\text{Eye}} = z_2(\mu) \frac{1}{n_f} \left[f_4(\mu\,;\mu_{\text{had.}}) - 1 \right] . \tag{6.13}$$

This eye contribution is essential since it provides the $\log\mu$ dependence which cancels with the ones from $z_6(\mu)f_6(\mu;\mu_{\rm had.})$ and $z_4(\mu)f_4(\mu;\mu_{\rm had.})$.[10] Note that this calculation is to our knowledge the first one where such a scale dependence cancellation in the Q_2-$Q_{4,6}$ mixing sector is explicitly shown.

Altogether, and at the $\mathcal{O}(N_c)$ we are working, we get

$$\begin{aligned} \mathrm{Re}\,\mathbf{g}_{\underline{8}} &= z_1(\mu)\left(-1+\frac{3}{5}g_{\Delta S=2}(\mu)\right)+z_2(\mu)\left(1-\frac{2}{5}g_{\Delta S=2}(\mu)\right)+ \\ &\quad + z_2(\mu)\frac{1}{n_f}\left[f_4(\mu\,;\mu_{\rm had.})-1\right]+z_6(\mu)f_6(\mu\,;\mu_{\rm had.})+z_4(\mu)f_4(\mu\,;\mu_{\rm had.})\,. \end{aligned} \tag{6.14}$$

The relation in eq. (6.10) which fixes $\mathbf{g}_{\underline{27}}$ in terms of $g_{\Delta S=2}(\mu)$ has been known for a long time [34] and, in the chiral limit, it holds to all orders in the $1/N_c$ expansion. In our case, the numerical result for the invariant $\hat{B}_K$ factor [12, 16] obtained in the chiral limit

$$\hat{B}_K=\frac{3}{4}C_{\Delta S=2}(\mu)\times g_{\Delta S=2}(\mu)=0.36\pm0.15 \tag{6.15}$$

implies

$$\mathbf{g}_{\underline{27}}=0.29\pm0.12\,, \tag{6.16}$$

which is perfectly consistent with the experimental value given in eq. (6.6). This is reassuring, because, at the order of approximations we are working, it is a complete calculation. However, for phenomenological applications, it remains to be seen how this result will be modified in the presence of chiral corrections.

By contrast, the result for $\mathrm{Re}\,\mathbf{g}_{\underline{8}}$ in eq. (6.14), is not yet a full $\mathcal{O}(N_c)$ calculation. We think, however, that it is worthwhile to present the numerical results which one already obtains at this level. There is no contribution to $\mathrm{Re}\,\mathbf{g}_{\underline{8}}$ from the $Q_{4,6}$ operators for $\mu\geq m_c$. Therefore, at $\mu=m_c\simeq1.3\,\mathrm{GeV}$, only the first line in eq. (6.14) is nonvanishing. The corresponding numerical results we obtain for $\mathrm{Re}\,\mathbf{g}_{\underline{8}}|_{Q_2,\mathrm{Eye}}$, for two input values of $|\langle\bar\psi\psi\rangle(\mu=2\,\mathrm{GeV})|$ (see discussion below) and letting the *large-N_c value* of L_5 vary in the range $1\times10^{-3}\leq L_5|_{\text{large-}N_c}\leq2\times10^{-3}$, have been tabulated in table 1 below. For each entry, the range of the results is the one corresponding to figure 6 by varying $\mu_{\rm had.}$ in the interval $0.8\,\mathrm{GeV}\lesssim\mu_{\rm had.}\lesssim1.3\,\mathrm{GeV}$, while, at the same time, allowing for violations of factorization in the residue of the OPE in eq. (4.20) by an extra factor of 2. This extra factor should take into account the fact that we are finding large deviations from the factorization of matrix elements of four-quark operators and, therefore, there could be large corrections also in the residues of the OPE in eq. (4.20).

From these results, we conclude that a fair estimate of $\mathrm{Re}\,\mathbf{g}_{\underline{8}}$, at the $\mathcal{O}(N_c)$ we have been working, lies in the range

$$\mathrm{Re}\,\mathbf{g}_{\underline{8}}=\underbrace{1.33\pm0.40}_{Q_2,Q_1}+\underbrace{0.8\pm0.4}_{Q_2-\mathrm{Eye}}=2.1\pm0.8\,. \tag{6.17}$$

[10] The μ scale dependence in $z_6(\mu)$ and $z_4(\mu)$ induced by the Q_2-$Q_{4,6}$ mixing (multiplied by the factorized piece of $f_{4,6}$ in eq. (6.12)) is cancelled by the μ scale dependence of f_4 in eq. (6.13).

$\lvert\langle\bar\psi\psi\rangle(\mu=2\,\mathrm{GeV})\rvert^{1/3}$	$L_5=1.0\times10^{-3}$	$L_5=1.5\times10^{-3}$	$L_5=2.0\times10^{-3}$
0.260 GeV	0.35 - 1.23	0.44 - 1.21	0.59 - 1.17
0.240 GeV	0.27 - 1.24	0.27 - 1.23	0.41 - 1.22

Table 1: Numerical results for $\mathrm{Re}\mathbf{g}_{\underline{8}}\big|_{Q_2,\mathrm{Eye}}$ for various input choices (see text).

In spite of the large errors involved, we find this result rather encouraging. When compared to the value $|\mathbf{g}_{\underline{8}}|_{\mathrm{exp.}}\sim 3.3$ to be explained, it certainly points in the right direction towards a dynamical understanding of the $\Delta I=1/2$ enhancement.

7. Phenomenology of ϵ'/ϵ

In terms of the isospin amplitudes A_0 and A_2 in eqs. (6.1) and (6.2), and to a very good approximation, one can write the CP violation observable ε'/ε as follows,

$$\frac{\varepsilon'}{\varepsilon}=e^{i\Phi}\frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right],\tag{7.1}$$

with

$$\Phi=\delta_2-\delta_0+\frac{\pi}{4}\simeq 0\,,\qquad\text{and}\qquad \omega=\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\,.\tag{7.2}$$

Using now the effective weak hamiltonian in eq. (1.3), one can obtain a formal expression of ε'/ε in terms of weak matrix elements of the four-quark operators Q_i as follows [4]

$$\frac{\varepsilon'}{\varepsilon}=\mathrm{Im}\,(V_{ts}^{*}V_{td})\;\frac{G_{\mathrm{F}}\omega}{2|\varepsilon||\mathrm{Re}A_0|}\;\left[P^{(0)}(1-\Omega_{\mathrm{IB}})-\frac{1}{\omega}P^{(2)}\right],\tag{7.3}$$

where

$$P^{(I)}=\sum_i y_i(\mu)\langle(\pi\pi)_I|Q_i(\mu)|K^0\rangle\,,\qquad\text{for}\qquad I=0,2\,,\tag{7.4}$$

and Ω_{IB} is a term induced by the effect of isospin breaking ($m_u\neq m_d$)

$$\Omega_{\mathrm{IB}}=\frac{1}{\omega}\frac{(\mathrm{Im}A_2)_{\mathrm{IB}}}{\mathrm{Im}A_0}\,.\tag{7.5}$$

It turns out that, because of the $\Delta I=1/2$ enhancement factor $1/\omega$, in front of $P^{(2)}$ on the r.h.s. of eq. (7.3), and because of the values of the Wilson coefficients $y_i(\mu)$, the two $P^{(I)}$ factors can be approximated, to a sufficiently good accuracy, as follows

$$P^{(0)}\simeq y_6(\mu)\langle(\pi\pi)_0|Q_6(\mu)|K^0\rangle\;+\;y_4(\mu)\langle(\pi\pi)_0|Q_4(\mu)|K^0\rangle\,,\tag{7.6}$$

and

$$P^{(2)}\simeq y_8(\mu)\langle(\pi\pi)_2|Q_8(\mu)|K^0\rangle\,,\tag{7.7}$$

where Q_8 denotes the electroweak penguin operator

$$Q_8=-12\sum_{q=u,d,s}e_q(\bar s_L q_R)(\bar q_R d_L)\,,\tag{7.8}$$

with e_q the electric charge of the quark q in e units.

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Recall that, since τ in eq. (6.11) is complex, the imaginary part of C_6 and C_4, and hence $\mathrm{Im}\mathbf{g}_{\underline{8}}|_{Q_6,Q_4}$ is proportional to y_6 and y_4. Our calculation of $g_{\underline{8}}|_{Q_6,Q_4}$ in section 5, allows, therefore, for an evaluation of $P^{(0)}$ at the corresponding approximation i.e., lowest order in χPT and next-to-leading leading order in the $1/N_c$ expansion, including terms of $\mathcal{O}(\frac{n_f}{N_c})$, with the result

$$P^{(0)} \simeq \sqrt{2}F_0(M_K^2 - m_\pi^2)\left[y_6(m_c)f_6(m_c\,;\mu_{\mathrm{had.}}) + y_4(m_c)f_4(m_c\,;\mu_{\mathrm{had.}})\right], \tag{7.9}$$

with the long-distance factors $f_6(m_c\,;\mu_{\mathrm{had.}})$ and $f_4(m_c\,;\mu_{\mathrm{had.}})$ given in eqs. (5.2) and (5.3).

We can also obtain an estimate of $P^{(2)}$ from our previous work in ref. [13]. As discussed there, to lowest $\mathcal{O}(p^0)$ in the chiral expansion, the four-quark operator Q_8 bosonizes as follows

$$\langle Q_8\rangle|_{\mathcal{O}(p^0)} = -12\,\underbrace{\langle O_2(\mu)\rangle}_{\langle 0|(\bar{s}_L s_R)(\bar{d}_R d_L)|0\rangle}\,\mathrm{tr}\left(U\lambda_L^{(23)}U^\dagger Q_R\right)^\dagger, \tag{7.10}$$

where $\lambda_L^{(23)} = \delta_{i2}\delta_{j3}$ and $Q_R = \mathrm{diag}[(2/3, -1/3, -1/3]$. The vev $\langle O_2(\mu)\rangle$ also appears in the Wilson coefficient of the $1/Q^6$ term in the OPE of the $\Pi_{LR}(Q^2)$ correlation function

$$\int d^4x\; e^{iq\cdot x}\langle 0|T\left(\bar{u}_L\gamma^\mu d_L(x)\bar{u}_R\gamma^\nu d_R(0)^\dagger\right)|0\rangle = \frac{1}{2i}(q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_{LR}(Q^2)\,, \tag{7.11}$$

for which the MHA to large-N_c QCD gives a rather good approximation, as discussed e.g. in ref. [10]. This offers the possibility of obtaining an estimate of the vev $O_2(\mu)$ beyond the strict large-N_c approximation, where $O_2 \Rightarrow \frac{1}{4}\langle\bar{\psi}\psi\rangle^2$, and, therefore, without having to fix a value for the $\langle\bar{\psi}\psi\rangle$ condensate, which is poorly known at present. To lowest order in the chiral expansion ($\mathcal{O}(p^0)$ in this case as seen from eq. (7.10)) we find [37, 38]

$$P^{(2)} \simeq y_8(\mu)\left(\frac{8}{3F_0^3}\right)F_0^2\frac{M_V^2M_A^2}{16\pi\alpha_{\mathrm{s}}(\mu)} \times \left[1 - \frac{\alpha_{\mathrm{s}}(\mu)}{\pi}\begin{pmatrix}25/8\\21/8\end{pmatrix}\begin{matrix}\text{NDR scheme}\\\text{HV scheme}\end{matrix}\right]. \tag{7.12}$$

We have now all the ingredients for a numerical evaluation of ϵ'/ϵ in eq. (7.3). For that purpose, we shall use the values of the physical parameters given in appendix A. By far, the most sensitive parameter in our determination of ε'/ε is the QCD quark condensate. Low values ($\langle\bar{\psi}\psi\rangle^{1/3} \sim -240\,\mathrm{MeV}$) are favoured by various QCD sum rules determinations, like e.g. the determination in ref. [36], using τ-data. High values ($\langle\bar{\psi}\psi\rangle^{1/3} \sim -260\,\mathrm{MeV}$) are favoured, however, by some of the lattice QCD simulations [39]. For two recent determinations see, however, ref. [40]. We, therefore, restrict the input value of $\langle\bar{\psi}\psi\rangle$ to a range

$$|\langle\bar{\psi}\psi\rangle(\mu = 2\,\mathrm{GeV})|^{1/3} = (250 \pm 10)\,\mathrm{MeV}\,. \tag{7.13}$$

Let us recall that the dependence of ϵ'/ϵ on $\langle\bar{\psi}\psi\rangle$ appears in the term $P^{(0)}$, trough the bosonization of the Q_6 operator i.e., the function $f_6(\mu;\mu_{\mathrm{had.}})$ in eq. (5.2), and it is the *sixth power* of $\langle\bar{\psi}\psi\rangle^{1/3}$ which counts!

Another important input parameter is the low-energy constant L_5. As we have already discussed, the L_5 which appears in the factorized contribution induced by the Q_6 operator has to be taken as running, while L_5 in the unfactorized contribution is constant, at the

$\lvert\langle\bar{\psi}\psi\rangle(\mu=2\,\mathrm{GeV})\rvert^{1/3}$	$P^{(0)}(1-\Omega_{\mathrm{IB}})\times 10^2$	$\frac{1}{\omega}P^{(2)}\times 10^2$	$\frac{\varepsilon'}{\varepsilon}\times 10^3$
0.260 GeV	8.1 - 9.2	3.9	2.0 - 3.4
0.250 GeV	6.3 - 7.2	3.9	1.2 - 2.1
0.240 GeV	5.0 - 5.6	3.9	0.5 - 1.1

Table 2: Numerical results for $P^{(0)}(1-\Omega_{\mathrm{IB}})$ in GeV^3, $(1/\omega)P^{(2)}$ in GeV^3 and ε'/ε for various input choices of $\langle\bar{\psi}\psi\rangle$. Here $1\times 10^{-3}\leq L_5|_{\text{large-}N_c}\leq 2\times 10^{-3}$ and $0.8\,\mathrm{GeV}\leq\mu_{\mathrm{had.}}\leq 1.3\,\mathrm{GeV}$.

level of accuracy that we are working in the $1/N_c$-expansion. As we have done in the calculation of $\mathrm{Reg}\underline{\mathbf{8}}|_{Q_2,\mathrm{Eye}}$ reported above, we leave $L_5|_{\text{large-}N_c}$ vary in the range $1\times 10^{-3}\leq L_5|_{\text{large-}N_c}\leq 2\times 10^{-3}$.

Concerning the factor Ω_{IB}, we have nothing to add at present. We recall, however, that the estimate has changed from

$$\Omega_{\mathrm{IB}}^{\pi^0\eta}=0.25\,,\quad \text{See ref. [41]} \tag{7.14}$$

to the more recent one χPT estimate [42]

$$\Omega_{\mathrm{IB}}^{\pi^0\eta}=0.16\pm 0.03\,, \tag{7.15}$$

which is the value that we shall be using here. We then have that

$$\frac{\varepsilon'}{\varepsilon}=\underbrace{\mathrm{Im}\,(V_{ts}^{*}V_{td})\;\frac{G_{\mathrm{F}}\omega}{2|\varepsilon||\mathrm{Re}A_0|}}_{(0.055\pm 0.008)\times\mathrm{GeV}^{-3}}\left[P^{(0)}\underbrace{(1-\Omega_{\mathrm{IB}})}_{0.84\pm 0.03}-\underbrace{\frac{1}{\omega}}_{22.2}\;P^{(2)}\right]. \tag{7.16}$$

We find large contributions for both $P^{(0)}$, mostly induced by the Q_6 operator (the contribution from the Q_4 operator amounts to less than 3% of the total), and for $P^{(2)}$. Typical numerical values are shown in Table 2 below for various input values of the quark condensate. The values in Table 2 have been obtained by letting $L_5|_{\text{large-}N_c}$ and $\mu_{\mathrm{had.}}$ vary in the ranges $1\times 10^{-3}\leq L_5|_{\text{large-}N_c}\leq 2\times 10^{-3}$ and $0.8\,\mathrm{GeV}\leq\mu_{\mathrm{had.}}\leq 1.3\,\mathrm{GeV}$. The numbers correspond to the renormalization scale $\mu=m_c=1.3\,\mathrm{GeV}$. The variations induced by the choice $0.8\,\mathrm{GeV}\leq\mu\leq 2\,\mathrm{GeV}$ (in $P^{(2)}$), or $0.7\,\mathrm{GeV}\leq M_S\leq 1.1\,\mathrm{GeV}$, change these numbers by 20%.

The results in table 2 show that, for the range of values of the quark condensate $\langle\bar{\psi}\psi\rangle$ in eq. (7.13), the values we obtain for ε'/ε are perfectly compatible with the latest world average [43] from the NA31, NA48 and KTeV experiments

$$\mathrm{Re}(\epsilon'/\epsilon)|_{\mathrm{Exp.}}=(1.66\pm 0.16)\times 10^{-3}\,. \tag{7.17}$$

A vanishing or negative value of ε'/ε, as reported by the lattice QCD groups [44, 45], appears to us very unlikely.

For the purpose of comparison with other theoretical predictions, it is convenient to represent eq. (7.16) as a straight line in a plane with $\frac{1}{\omega}P^{(2)}$ plotted in the horizontal axis, and $P^{(0)}(1-\Omega_{\mathrm{IB}})$ in the vertical axis [46]. This is the plot shown in figure 8, where the

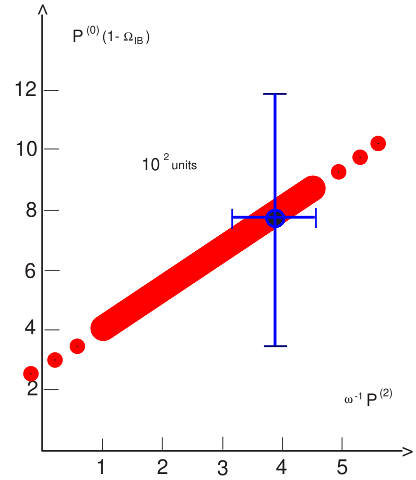

Figure 8: The experimental value of ϵ'/ϵ in eq. (7.17) fixes the straight line in the the figure, with a certain width due to experimental errors, see eq. (7.16). Our theoretical prediction, including errors, is represented by the cross with error bars.

width of the solid line reflects the experimental errors in the overall factor in the r.h.s. of eq. (7.16) and in eq. (7.17). The cross in the same plot, represents the prediction which follows from our present estimates of $P^{(0)}$ and $P^{(2)}$,[11] for the restricted input of the quark condensate in eq. (7.13), but including the errors of scanning the other input parameters in eqs. (A.5) to (A.6) of the appendix, and varying the matching scale between 800 MeV and 2 GeV. Furthermore we have also allowed for violations of factorization in the residue of the OPE in eq. (4.19) by a factor of 2, as we have done before for eq. (4.20). It turns out, however, that the influence of a possible error in this residue is now much milder because the UV behaviour of the integral of $Q^2\mathcal{W}_{\rm DGRR}(Q^2)$ is very much dominated by the Goldstone double pole term.

8. Discussion and conclusion

We hope to have shown how one can treat analytically, and on a sound theoretical basis, the calculation of electroweak matrix elements beyond factorization. In particular, one important goal of this work was to study the magnitude of the unfactorized contributions in two cases of special interest, i.e. ϵ'/ϵ and the $\Delta I = 1/2$ rule. In section 4, we explained how, bringing new hadronic scales, the unfactorized contributions may turn out to be sizeable as compared to the factorized ones, depending on the scales involved. Already at the level of the leading $\mathcal{O}(n_f/N_c)$ corrections, we have shown by an explicit calculation that this is indeed the case for the Q_6 and Q_4 penguin operators.

For the operator Q_6, the unfactorized contribution turns out to be even much larger than its factorized one, due to the large chiral coefficient $\sim L_5 - \frac{5}{2}L_3$ in eq. (4.17) which through eq. (4.28) brings a large coefficient to the hadronic scales in eq. (5.2). As a result,

[11] The error in $P^{(2)}$ does not include other analytic determinations [49]–[52] and lattice QCD determinations [53, 54, 44, 45].

the leading piece is not the factorized contribution but the unfactorized one for which we just calculated the lowest order contribution. From this leading lowest order contribution, the higher order corrections should now be calculated in subsequent works. Similarly for Q_4 (and for the directly related eye contribution of Q_2), we find a rather large unfactorized contribution through eqs. (4.29) and (5.3).

Note that the claim that unfactorized contributions are large was already made in refs. [17, 19, 21, 23] for $Q_{1,2}$ and in refs. [18, 20, 22] for Q_6. In refs. [18, 20], the calculation of the unfactorized contribution was done for Q_6 by just taking the effects of the pseudoscalar mesons regularizing the pseudoscalar loops with an euclidean cut-off Λ_c. From our present analysis, which incorporates the effect of hadronic resonances explicitly, we observe that this can be justified at the qualitative level, because, as was anticipated in those references, a quadratically divergent ($\sim \Lambda_c^2$) piece can be interpreted as the reflect of terms proportional to the square of the masses of the hadronic resonances. In our more refined approach, the cut-off Λ_c^2 is replaced by combinations of M_V^2, M_S^2, $M_{P'}^2$ and $\mu_{\text{had.}}^2$. This is also what happens for the quadratic coefficient of the Q_6 matrix element (i.e. the r.h.s of eq. (4.28) proportional to $L_5-\frac{5}{2}L_3$).[12] However, there is no way, with the cut-off procedure advocated in refs. [17]–[20], to fix unambiguously the choice of Λ_c, and there is no control at all on the short-distance matching scale dependence. The unfactorized contribution has also been estimated in refs. [47] within the constituent chiral quark model of ref. [23], again without short-distance matching. In refs. [21, 22], the previous problem is somewhat ameliorated, although within the framework of the ENJL model. It turns out, however, that in this model it is also found that the unfactorized contributions are large.

The advantage of the large-N_c QCD approach presented here is that it allows to use the same dimensional regularization both at short and long distances. We have been able to do a fully analytic matching of scale dependences, which allows for a clear separation of the various scales and hadronic quantum numbers involved in the problem.

To improve our calculation requires the incorporation of several effects which are beyond the scope of the present analysis. The $\mathcal{O}(1/N_c)$ not enhanced by a n_f factor should be calculated to have a full NLO calculation in the $1/N_c$ expansion. The long-distance scheme dependence should also be calculated to cancel the corresponding short-distance one (which can be done exactly in our framework as was shown for B_K in ref. [12]). The hadronic ansatz could be refined, systematically, by considering higher dimensional short- and long- distance constraints. Moreover, the effect of the final state interactions (FSI) could eventually be incorporated, following the work in ref. [48]. Notice, however, that the effect of the unfactorized contributions which we find is much larger than the size of the FSI effects estimated in this reference.[13]

Finally, we observe that a precise calculation of ε'/ε, within the framework we have discussed here, is correlated with the determination of some of the low energy constants

[12] Note however that as was already pointed out above the important L_3 contribution was missing in this quadratic coefficient in ref. [20]. As a result the enhancement of the Q_6 contribution to ε'/ε found in ref. [20] was smaller than the one we obtain here.

[13] In refs.[48], the unfactorized contribution accounts for an error of $\pm 0.5 \cdot 10^{-3}$ in their quoted value of ε'/ε. We find here that this is an underestimate.

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entering our input; in particular the value of the quark condensate $\langle\bar{\psi}\psi\rangle$ and to a lesser extent the L_5 coupling. Other important input values are Ω_{IB} (where it would be nice to include as well the unfactorized contribution which, so far, has been always neglected), Λ_{QCD} and Im $V_{ts}V_{td}^*$.

As a result of all the considerations discussed above, we believe that a systematical analytic calculation of ε'/ε in the Standard Model is now becoming conceivable although, in our opinion, matching the level of the experimental precision is likely to become a very difficult task.

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A. Compilation of numerical inputs

The numerical input we have used in the text are as follows[14]:

$$\mathrm{Im}\,(V_{ts}^*V_{td})\;\frac{G_{\mathrm{F}}\omega}{2|\varepsilon||\mathrm{Re}A_0|} = 0.055 \pm 0.008\,, \qquad \text{ref. [35].} \tag{A.1}$$

$$\begin{aligned}
M_W &= 80.4\,\mathrm{GeV}\,, & \sin^2\theta_W &= 0.23\,, & \alpha &= \frac{1}{137.036}\,,\\
m_c &= 1.3\,\mathrm{GeV}\,, & m_b &= 4.4\,\mathrm{GeV}\,, & m_t &= 170\,\mathrm{GeV}\,,\\
M_K &= 0.498\,\mathrm{GeV}\,, & m_\pi &= 0.135\,\mathrm{GeV}\,, & \omega &= 1/22.2\,,\\
F_0 &= 0.087\,\mathrm{GeV}\,, & M_V &= 0.770\,\mathrm{GeV}\,, & M_S &= 0.9\pm 0.2\,\mathrm{GeV}\,,\\
M_A &= 1.2\,\mathrm{GeV}\,, & M_{V'} &= 1.4\,\mathrm{GeV}\,, & M_{P'} &= 1.3\,\mathrm{GeV}\,,\\
L_3 &= -3.0\cdot 10^{-3}\,, & L_5(M_\rho) &= 1.4\cdot 10^{-3}\,, & L_9(M_\rho) &= 6.9\cdot 10^{-3}\,.
\end{aligned} \tag{A.2}$$

and

$$\langle\bar{\psi}\psi\rangle(\mu = 2\,\mathrm{GeV}) = -(250 \pm 10\,\mathrm{MeV})^3\,, \tag{A.3}$$

$$L_5|_{\text{large-}N_c} = (1.5 \pm 0.5)\times 10^{-3} \tag{A.4}$$

$$\Lambda^{(4)}_{\overline{\mathrm{MS}}} = (325 \pm 40)\,\mathrm{MeV}\,, \tag{A.5}$$

$$\Omega_{IB} = 0.16 \pm 0.03\,. \tag{A.6}$$

[14] Notice that the A_0 amplitude in ref. [4] is *ours* times a factor $\sqrt{3/2}$.

The Wilson coefficients we have used have been calculated using the same program as the one which has been used in ref. [4]. For all calculations we used LO Wilson coefficient except for the evaluation of $P^{(2)}$ in eq. (7.12) for which we took the NLO-NDR Wilson coefficients.

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